VIZSGA 2023.07.04. MEGOLDÁSOK

1. feladat (6p)

$$z^3+4z^2+6z = z\cdot(z^2+4z+6)$$ (1p)

$$z_1 = 0 \qquad z_{2,3} = \frac{-4 \pm \sqrt{16-4\cdot 6}}{2} =$$

(1p)

$$= \frac{-4 \pm \sqrt{-8}}{2} = -2 \pm \sqrt{2}\cdot i$$ (2p)

Szorzatalak

$$z^3+4z^2+6z = z\cdot\left(z-(-2-\sqrt{2}i)\right)\left(z-(-2+\sqrt{2}i)\right)$$

(2p)

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(2p)

2. feladat (6p)

$P(1,2,1)$

$\underline{n}(3,-3,0)$

$\Sigma_1: \quad 3(x-1)-3(y-2)+0(z-1)=0$

$3x-3y=-3$ 2p

$Q(0,0,4)$

$d(\Sigma_1, Q) = \left| \frac{3x-3y+3}{|\underline{n}|} \right|_Q =$ 2p

$= \left| \frac{3\cdot 0 - 3\cdot 0 + 3}{\sqrt{3^2+(-3)^2}} \right| = \frac{3}{\sqrt{18}} = \frac{1}{\sqrt{2}}$

2p

3. feladat (8p)

$$\left.\begin{aligned} x+y-z&=1\\ 2x-y+c\cdot z&=2\\ 4x-y+z&=1 \end{aligned}\right\} \qquad \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1\\ 2 & -1 & c & 2\\ 4 & -1 & 1 & 1 \end{array}\right) \underset{\sim}{\underline{s_2}\leftrightarrow \underline{s_3}}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1\\ 4 & -1 & 1 & 1\\ 2 & -1 & c & 2 \end{array}\right) \underset{\underline{s_3}-2\underline{s_1}}{\overset{\underline{s_2}-4\underline{s_1}}{\sim}} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1\\ 0 & -5 & 5 & -3\\ 0 & -3 & c+2 & 0 \end{array}\right) \underset{\sim}{\tfrac{1}{5}\underline{s_2}}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1\\ 0 & -1 & 1 & -3/5\\ 0 & -3 & c+2 & 0 \end{array}\right) \underset{\sim}{\underline{s_3}-3\underline{s_2}} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1\\ 0 & -1 & 1 & -3/5\\ 0 & 0 & c-1 & 9/5 \end{array}\right)$$ (3p)

Ha $c=1 \Rightarrow 0 \neq 9/5$ ellentmondás, nincs m.o. (2p)

Ha $c \neq 1 \Rightarrow z = \dfrac{9}{5c-5}$; $y = z + \dfrac{3}{5} = \dfrac{9}{5c-5} + \dfrac{3}{5} = \dfrac{6-3c}{5c-5}$

$x = 1+z-y = 1+z-z-\dfrac{3}{5} = \dfrac{2}{5}$ (3p)

4. feladat (9p)

$$f(x,y) = \frac{x^3 - y^3}{xy} = \frac{x^2}{y} - \frac{y^2}{x}$$

$$f'_x = \frac{2x}{y} + \frac{y^2}{x^2}$$

$$f'_y = -\frac{x^2}{y^2} - \frac{2y}{x}$$

$$f(1,2) = \frac{-7}{2}$$

1p

$$\underline{\text{grad}}\, f(1,2) = \left(\frac{2\cdot 1}{2} + \frac{2^2}{1^2} ; -\frac{1^2}{2^2} - \frac{2\cdot 2}{1} \right) = \left(5 ; -\frac{17}{4} \right)$$

2p

$$\underline{v}^0 = \frac{\underline{v}}{|\underline{v}|} = \frac{(3,4)}{\sqrt{9+16}} = \left(\frac{3}{5} ; \frac{4}{5} \right)$$

2p

$$f'_{\underline{v}}(1,2) = \left\langle \left(5, -\frac{17}{4}\right) ; \left(\frac{3}{5}, \frac{4}{5}\right) \right\rangle = 3 - \frac{17}{5} = -\frac{2}{5} =$$

$$= -0{,}4$$

1p

1p

Érintősík:

$$z = 5(x-1) - \frac{17}{4}(y-2) - \frac{7}{2}$$

$$5x - 17/4\, y - z = 0$$

2p

5. feladat (8p)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+n}}$$

- alternáló
- $\lim_{n\to\infty} \frac{(-1)^n}{\sqrt{n^2+n}} = 0$
- $|a_{n+1}| < |a_n|$

$$\frac{1}{\sqrt{(n+1)^2+n+1}} < \frac{1}{\sqrt{n^2+n}}$$

$$n^2+n < n^2+3n+2$$

monoton csökkenő

Leibniz típusú tehát konvergens (3p)

Abszolút sora:

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}}$$ (1p)

$$\frac{1}{\sqrt{n^2+n}} > \frac{1}{\sqrt{2n^2}} = \frac{1}{\sqrt{2}} \cdot \frac{1}{n}$$ (2p)

mivel $\frac{1}{\sqrt{2}} \cdot \sum_{n=1}^{\infty} \frac{1}{n}$ divergens minoráns, ezért az eredeti sor feltételesen konvergens (nem abszolút konvergens) (2p)

6. feladat (8p)

$f(x) = \sin(x^2)$ tudjuk, hogy $\sin(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)!} x^{2n-1}$ (1p)

$\forall x \in \mathbb{R}$ esetén, ekkor

$$\sin(x^2) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)!} (x^2)^{2n-1} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)!} \cdot x^{4n-2} =$$ (2p)

$$= x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \dots$$ ha $x^2 \in \mathbb{R}$ azaz $\forall x \in \mathbb{R}$

(1p)

Tehát a konvergenciasugár $R = \infty$

$$\sin(1) \approx 1^2 - \frac{1^6}{3!} + \frac{1^{10}}{5!} \dots$$

(2p)

A Taylor-sor Leibniz, így a hiba $\left| \frac{1}{(2n-1)!} \right| < \frac{1}{100}$

azaz $100 < (2n-1)!$ (1p)

$3! = 6 \quad 5! = 120$

$\sin(1) \approx 1 - \frac{1}{6} = \frac{5}{6} \approx 0{,}8\dot{3}$

(1p)