1. zárthelyi

1. $$\int_{-2}^{2} \frac{3}{\sqrt{2-x}}\,dx = \lim_{\varepsilon\to 0+} \int_{-2}^{2-\varepsilon} \frac{3}{\sqrt{2-x}}\,dx = \lim_{\varepsilon\to 0+} \left[-\frac{3\sqrt{2-x}}{\frac{1}{2}}\right]_{-2}^{2-\varepsilon} =$$

$$= \lim_{\varepsilon\to 0+} -6\sqrt{2-(2-\varepsilon)} + 6\sqrt{2-(-2)} = \lim_{\varepsilon\to 0+} -6\underbrace{\sqrt{\varepsilon}}_{\to 0} + 6\sqrt{4} = 12$$

2. $z_1 = -2+i$, $z_2 = 1+3i$

$$z = \frac{\overline{z_1 \cdot z_2}}{2z_1 - z_2}$$

$$\overline{z_1 \cdot z_2} = \overline{(-2+i)(1+3i)} = \overline{(-2-3)+(1-6)i} = -5+5i$$

$$2z_1 - z_2 = -4+2i-1-3i = -5-i$$

$$z = \frac{-5+5i}{-5-i} = \frac{(-5+5i)(-5+i)}{26} = \frac{(25-5)+(-25-5)i}{26} = \frac{10}{13} - \frac{15}{13}i$$

3. $P(3,4,1)$, $\underline{n}(2,-1,2)$ $\Big\}$ α sík: $2(x-3)-(y-4)+2(z-1)=0$

$$2x-y+2z-4=0$$

$Q(-1,3,-5)$ normálalak: $|\underline{n}| = \sqrt{2^2+(-1)^2+2^2} = \sqrt{9} = 3$

$$d(\alpha, Q) = \left|\frac{2\cdot(-1)-3+2\cdot(-5)-4}{3}\right| = \left|\frac{-2-3-10-4}{3}\right| = \frac{19}{3}$$

4.

$$\underline{\underline{A}}^T \cdot \underline{\underline{B}} = \begin{pmatrix} 1 & -2 \\ 0 & -3 \\ 4 & 3 \end{pmatrix} = \underline{\underline{B}}$$

$$\underline{\underline{A}}^T = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & -3 \\ 7 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -3 \\ 7 & 5 \end{pmatrix}$$

$$\left(\underline{\underline{A}}^T\underline{\underline{B}}\right)^2 = \begin{pmatrix} 3 & -3 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} -12 & -24 \\ 56 & 4 \end{pmatrix}$$

2. zárthelyi

1.

$$\det(\underline{\underline{A}}) = \begin{vmatrix} 4 & -2 & 1 \\ -2 & 0 & -3 \\ 3 & p & 4 \end{vmatrix} = -(-2)\begin{vmatrix} -2 & 1 \\ p & 4 \end{vmatrix} - (-3)\begin{vmatrix} 4 & -2 \\ 3 & p \end{vmatrix} = 2(-8-p) + 3(4p+6) =$$

$$= -16 - 2p + 12p + 18 = 10p + 2 \qquad p = \frac{-2}{10} = \frac{-1}{5} \text{-re}$$

szinguláris a mátrix

2.

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -2 & 0 & -3 & 0 \\ -4 & 0 & 2 & 1 & 0 \end{array}\right) \underset{\underline{s}_3 + 4\underline{s}_1}{\overset{\underline{s}_2 - \underline{s}_1}{\sim}} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -3 & -1 & -4 & 0 \\ 0 & 4 & 6 & 5 & 0 \end{array}\right) \overset{\underline{s}_2 + \underline{s}_3}{\sim} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 5 & 1 & 0 \\ 0 & 4 & 6 & 5 & 0 \end{array}\right) \overset{\underline{s}_3 - 4\underline{s}_2}{\sim}$$

$$\overset{\underline{s}_3 - 4\underline{s}_2}{\sim} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 5 & 1 & 0 \\ 0 & 0 & -14 & 1 & 0 \end{array}\right)$$

$\text{rang}(\underline{\underline{A}}) = \text{rang}(\underline{\underline{A}} | \underline{b}) = 3 < n = 4$

∞ sok megoldás

$$\Rightarrow \quad -14z + u = 0 \qquad y + 5z + u = 0 \qquad x + y + z + u = 0$$

$$\boxed{\begin{array}{c} u = 14z \\ z \in \mathbb{R} \end{array}}$$

$$y = -5z - u = -5z - 14z$$

$$\boxed{y = -19z}$$

$$x = -y - z - u$$

$$x = 19z - z - 14z$$

$$\boxed{x = 4z}$$

3.

$$\underline{\underline{A}} = \begin{pmatrix} 3 & 4 & -2 \\ 1 & -2 & -1 \\ 2 & -1 & -1 \end{pmatrix} \qquad \det(\underline{\underline{A}}) = 3(\overset{3}{2-1}) - 4(\overset{-4}{-1+2}) - 2(\overset{-6}{-1+4}) = $$

$$= -7 \quad (\neq 0 \text{ OK})$$

$$\underline{b} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \qquad y = \frac{\begin{vmatrix} 3 & 1 & -2 \\ 1 & 3 & -1 \\ 2 & -1 & -1 \end{vmatrix}}{-7} = \frac{3\cdot(-4) - 1\cdot 1 - 2(-7)}{-7} = -\frac{1}{7}$$

4.

$$\begin{vmatrix} -1-\lambda & 3 \\ 3 & -1-\lambda \end{vmatrix} = (1+\lambda)^2 - 9 = \lambda^2 + 2\lambda - 8 = (\lambda+4)(\lambda-2) \qquad \begin{array}{l} \lambda_1 = -4 \\ \lambda_2 = 2 \end{array}$$

$$\lambda_1 = -4 \qquad \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{c} 3x + 3y = 0 \\ x = -y \end{array} \qquad \underline{s}_{-4} = y\begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad y \in \mathbb{R} \setminus \{0\}$$

$$\lambda_2 = 2 \qquad \begin{pmatrix} -3 & 3 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{c} -3x + 3y = 0 \\ x = y \end{array} \qquad \underline{s}_2 = y\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad y \in \mathbb{R} \setminus \{0\}$$

Sajátbázis: $\left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$; $\underline{\underline{B}} = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$; $\underline{\underline{B}}^{-1} = \frac{1}{-2}\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$

$$\begin{array}{ccc} & \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix} & \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \\ \underline{\underline{B}}^{-1}\underline{\underline{A}}\,\underline{\underline{B}} = \begin{pmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} & \begin{pmatrix} 2 & -2 \\ 1 & 1 \end{pmatrix} & \begin{pmatrix} -4 & 0 \\ 0 & 2 \end{pmatrix} \end{array}$$

Diagonális alak.