Top dense ball packings and coverings in 3-dimensional hyperbolic space

In an n-dimensional space of constant curvature $\mathbf{E}^n$, $\mathbf{H}^n$, $\mathbf{S}^n$ $(n \geq 2)$ let $d_n(r)$ be the density of $n+1$ spheres of radius r mutually touching one another with respect to the simplex spanned by the centres of the spheres. L. Fejes Tóth and H. S. M. Coxeter conjectured that in an n-dimensional space of constant curvature the density of packing spheres of radius r can not exceed $d_n(r)$. This conjecture has been proved by C. Roger in the Euclidean space. The 2-dimensional case has been solved by L. Fejes Tóth. In an 3-dimensional space of constant curvature the problem has been investigated by K. Böröczky and A. Florian and it has been studied by K. Böröczky for n-dimensional space of constant curvature $(n \geq 4)$. The upper bound $d_n(\infty)$ $(n = 2, 3)$ is attained for a regular horoball packing, that is, a packing by horoballs which are inscribed in the cells of a regular honeycomb of $\overline{\mathbf{H}}^n$. For dimensions $n = 2$, there is only one such packing. It belongs to the regular tessellation $\{\infty, 3\}$. Its dual $\{3, \infty\}$ is the regular tessellation by ideal triangles all of whose vertices are surrounded by infinitely many triangles. This packing has in-circle density $d_2(\infty) = \frac{3}{\pi} \approx 0.95493$.

In $\overline{\mathbf{H}}^3$ there is exactly one horoball packing with horoballs in same type whose Dirichlet–Voronoi cells give rise to a regular honeycomb described by the Schläfli symbol $\{6,\ 3,\ 3\}$. Its dual $\{3, 3, 6\}$ consists of ideal regular simplices T^{∞}_{reg} with dihedral angles $\frac{\pi}{3}$ building up a 6-cycle around each edge of the tessellation. The density of this packing is $\delta_3^{\infty} \approx 0.85328$

But, there are no "essential" results regarding the "classical ball packings and coverings" with congruent balls. What are the densest ball arrangements in $\mathbf{H}^n$ and what are their densities?

The goal of this talk to study the above problem of finding the densest ball arrangements in 3-dimensional hyperbolic space with "classical balls". In this talk we consider congruent periodic ball packings (for simplicity) related to the generalized Coxeter orthoschemes. We formulate a conjecture for the densest ball packing arrangement and compute its density.

We will use the well-known Beltrami-Cayley-Klein modell of $\mathbf{H}^3$ with projective metric calculus.