Non-local games and Bell inequalities

Dávid Bugár

Institute of Mathematics, Budapest University of Technology and Economics

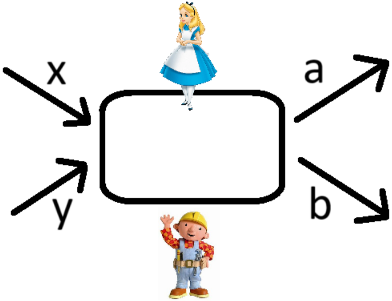
What's in the box?

- We can only infer $P(a|x)$

What's in the box?

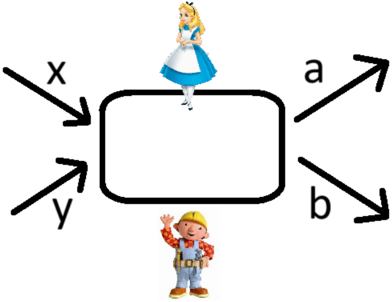
- We can only infer $P(a|x)$
- Is it classical/quantum?

Non-local games



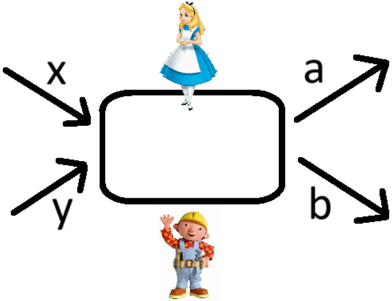
- Decision function: $\delta(x, y, a, b) = \begin{cases} 0 & \text{if they lost} \\ 1 & \text{if they won} \end{cases}$

Non-local games



- Decision function: $\delta(x, y, a, b) = \begin{cases} 0 & \text{if they lost} \\ 1 & \text{if they won} \end{cases}$
- $\pi(x, y)$ is the probability of getting the input pair x, y.

Non-local games



- Decision function: $\delta(x, y, a, b) = \begin{cases} 0 & \text{if they lost} \\ 1 & \text{if they won} \end{cases}$
- $\pi(x, y)$ is the probability of getting the input pair x, y.
- Alice and Bob can discuss the strategy before the game: $P(a, b|x, y)$.

CHSH game

- Clauser, Horne, Shimony, Holt

CHSH game

- Clauser, Horne, Shimony, Holt
- $a, b, x, y \in \{0, 1\}$

CHSH game

- Clauser, Horne, Shimony, Holt
- $a, b, x, y \in \{0, 1\}$
- distribution of inputs $\pi(x, y) = \frac{1}{4}$

CHSH game

- Clauser, Horne, Shimony, Holt
- $a, b, x, y \in \{0, 1\}$
- distribution of inputs $\pi(x, y) = \frac{1}{4}$
- $\delta(x, y, a, b) = 1 \Leftrightarrow a + b = xy \mod 2$

Strategies as stochastic matrices

- Alice and Bob wants to maximize the winning probability $\omega = \sum_{x,y,a,b} \delta(x, y, a, b) P(a, b|x, y) \pi(x, y)$.

Strategies as stochastic matrices

- Alice and Bob wants to maximize the winning probability $\omega = \sum_{x,y,a,b} \delta(x,y,a,b) P(a,b|x,y) \pi(x,y)$.

- The strategy $P(a,b|x,y)$ is a stochasitc 4x4 matrix:

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$P(0,0,0,0)$	$P(0,0,0,1)$	$P(0,0,1,0)$	$P(0,0,1,1)$
$(0,1)$	$P(0,1,0,0)$	$P(0,1,0,1)$	$P(0,1,1,0)$	$P(0,1,1,1)$
$(1,0)$	$P(1,0,0,0)$	$P(1,0,0,1)$	$P(1,0,1,0)$	$P(1,0,1,1)$
$(1,1)$	$P(1,1,0,0)$	$P(1,1,0,1)$	$P(1,1,1,0)$	$P(1,1,1,1)$
Sum	1	1	1	1

Strategies as stochastic matrices

- Alice and Bob wants to maximize the winning probability $\omega = \sum_{x,y,a,b} \delta(x,y,a,b) P(a,b|x,y) \pi(x,y)$.

- The strategy $P(a,b|x,y)$ is a stochasitc 4x4 matrix:

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$P(0,0,0,0)$	$P(0,0,0,1)$	$P(0,0,1,0)$	$P(0,0,1,1)$
$(0,1)$	$P(0,1,0,0)$	$P(0,1,0,1)$	$P(0,1,1,0)$	$P(0,1,1,1)$
$(1,0)$	$P(1,0,0,0)$	$P(1,0,0,1)$	$P(1,0,1,0)$	$P(1,0,1,1)$
$(1,1)$	$P(1,1,0,0)$	$P(1,1,0,1)$	$P(1,1,1,0)$	$P(1,1,1,1)$
Sum	1	1	1	1

- $0 \leq P(a,b|x,y) \leq 1$, $\sum_{a,b} P(a,b|x,y) = 1$ (hyperplanes)

Strategies as stochastic matrices

- Alice and Bob wants to maximize the winning probability $\omega = \sum_{x,y,a,b} \delta(x,y,a,b) P(a,b|x,y) \pi(x,y)$.

- The strategy $P(a,b|x,y)$ is a stochasitc 4x4 matrix:

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$P(0,0,0,0)$	$P(0,0,0,1)$	$P(0,0,1,0)$	$P(0,0,1,1)$
$(0,1)$	$P(0,1,0,0)$	$P(0,1,0,1)$	$P(0,1,1,0)$	$P(0,1,1,1)$
$(1,0)$	$P(1,0,0,0)$	$P(1,0,0,1)$	$P(1,0,1,0)$	$P(1,0,1,1)$
$(1,1)$	$P(1,1,0,0)$	$P(1,1,0,1)$	$P(1,1,1,0)$	$P(1,1,1,1)$
Sum	1	1	1	1

- $0 \leq P(a,b|x,y) \leq 1$, $\sum_{a,b} P(a,b|x,y) = 1$ (hyperplanes)
- Strategies live on the intersection of a 16 dimensional hypercube and 4 hyperplanes $\longrightarrow$ 12 dimensional polytope!

Strategies as stochastic matrices

- Alice and Bob wants to maximize the winning probability $\omega = \sum_{x,y,a,b} \delta(x,y,a,b) P(a,b|x,y) \pi(x,y)$.

- The strategy $P(a,b|x,y)$ is a stochasitc 4x4 matrix:

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$P(0,0,0,0)$	$P(0,0,0,1)$	$P(0,0,1,0)$	[redacted]
$(0,1)$	[redacted]	[redacted]	[redacted]	$P(0,1,1,1)$
$(1,0)$	[redacted]	[redacted]	[redacted]	$P(1,0,1,1)$
$(1,1)$	$P(1,1,0,0)$	$P(1,1,0,1)$	$P(1,1,1,0)$	[redacted]
Sum	1	1	1	1

- $0 \leq P(a,b|x,y) \leq 1$, $\sum_{a,b} P(a,b|x,y) = 1$ (hyperplanes)
- Strategies live on the intersection of a 16 dimensional hypercube and 4 hyperplanes $\longrightarrow$ 12 dimensional polytope!
- The decision function is a mask on the strategy matrix!

Non-signaling strategies

- Alice and Bob should not cheat!

Non-signaling strategies

- Alice and Bob should not cheat!
- Non-signaling condition:
  $\sum_a P(a,b|x,y)$ independent of x.
  $\sum_b P(a,b|x,y)$ independent of y.

Non-signaling strategies

- Alice and Bob should not cheat!
- Non-signaling condition:
  $\sum_a P(a,b|x,y)$ independent of x.
  $\sum_b P(a,b|x,y)$ independent of y.
- In CHSH this means:
  $P(0,b|0,y) + P(1,b|0,y) = P(0,b|1,y) + P(1,b|1,y)$
  for any b, y.
  $P(a,0|x,0) + P(a,1|x,0) = P(a,0|x,1) + P(a,1|x,1)$
  for any a, x.

Non-signaling strategies

- Alice and Bob should not cheat!
- Non-signaling condition:
  $\sum_a P(a, b|x, y)$ independent of x.
  $\sum_b P(a, b|x, y)$ independent of y.
- In CHSH this means:
  $P(0, b|0, y) + P(1, b|0, y) = P(0, b|1, y) + P(1, b|1, y)$
  for any b, y.
  $P(a, 0|x, 0) + P(a, 1|x, 0) = P(a, 0|x, 1) + P(a, 1|x, 1)$
  for any a, x.
- Geometrically: Hyperplanes!

Non-signaling strategies

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$P(0,0,0,0)$	$P(0,0,0,1)$	$P(0,0,1,0)$	$P(0,0,1,1)$
$(0,1)$	$P(0,1,0,0)$	$P(0,1,0,1)$	$P(0,1,1,0)$	$P(0,1,1,1)$
$(1,0)$	$P(1,0,0,0)$	$P(1,0,0,1)$	$P(1,0,1,0)$	$P(1,0,1,1)$
$(1,1)$	$P(1,1,0,0)$	$P(1,1,0,1)$	$P(1,1,1,0)$	$P(1,1,1,1)$
Sum	1	1	1	1

- Non-signaling condition:
  Sum in solid rectangle = Sum in dashed rectangle
  (each color separately)

Non-signaling strategies

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$P(0,0,0,0)$	$P(0,0,0,1)$	$P(0,0,1,0)$	$P(0,0,1,1)$
$(0,1)$	$P(0,1,0,0)$	$P(0,1,0,1)$	$P(0,1,1,0)$	$P(0,1,1,1)$
$(1,0)$	$P(1,0,0,0)$	$P(1,0,0,1)$	$P(1,0,1,0)$	$P(1,0,1,1)$
$(1,1)$	$P(1,1,0,0)$	$P(1,1,0,1)$	$P(1,1,1,0)$	$P(1,1,1,1)$
Sum	1	1	1	1

- Non-signaling condition:
  Sum in solid rectangle $=$ Sum in dashed rectangle
  (each color separately)

Non-signaling strategies

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$P(0,0,0,0)$	$P(0,0,0,1)$	$P(0,0,1,0)$	$P(0,0,1,1)$
$(0,1)$	$P(0,1,0,0)$	$P(0,1,0,1)$	$P(0,1,1,0)$	$P(0,1,1,1)$
$(1,0)$	$P(1,0,0,0)$	$P(1,0,0,1)$	$P(1,0,1,0)$	$P(1,0,1,1)$
$(1,1)$	$P(1,1,0,0)$	$P(1,1,0,1)$	$P(1,1,1,0)$	$P(1,1,1,1)$
Sum	1	1	1	1

- Non-signaling condition:
  Sum in solid rectangle = Sum in dashed rectangle
  (each color separately)
- Let's search NS strategies!

Deterministic strategies

- $P(a, b|x, y) \in \{0, 1\}$

Deterministic strategies

- $P(a, b|x, y) \in \{0, 1\}$
- Deterministic strategies = Vertices of the polytope of strategies

Deterministic strategies

- $P(a, b|x, y) \in \{0, 1\}$
- Deterministic strategies = Vertices of the polytope of strategies
- (Winning) Example:

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	0	1	0
$(0,1)$	0	0	0	1
$(1,0)$	0	0	0	0
$(1,1)$	0	1	0	0

Deterministic strategies

- $P(a, b|x, y) \in \{0, 1\}$
- Deterministic strategies = Vertices of the polytope of strategies
- (Winning) Example:

$(a, b) \setminus (x, y)$	$(0, 0)$	$(0, 1)$	$(1, 0)$	$(1, 1)$
$(0, 0)$	1	0	1	0
$(0, 1)$	0	0	0	1
$(1, 0)$	0	0	0	0
$(1, 1)$	0	1	0	0

- But this is signaling!

Deterministic strategies

- $P(a,b|x,y) \in \{0,1\}$
- Deterministic strategies = Vertices of the polytope of strategies
- (Winning) Example:

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	0	1	0
$(0,1)$	0	0	0	1
$(1,0)$	0	0	0	0
$(1,1)$	0	1	0	0

- But this is signaling!
- What are the deterministic non-signaling strategies?

Deterministic non-signaling strategies

Let's play SUDOKU!

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$P(0,0,0,0)$	$P(0,0,0,1)$	$P(0,0,1,0)$	$P(0,0,1,1)$
$(0,1)$	$P(0,1,0,0)$	$P(0,1,0,1)$	$P(0,1,1,0)$	$P(0,1,1,1)$
$(1,0)$	$P(1,0,0,0)$	$P(1,0,0,1)$	$P(1,0,1,0)$	$P(1,0,1,1)$
$(1,1)$	$P(1,1,0,0)$	$P(1,1,0,1)$	$P(1,1,1,0)$	$P(1,1,1,1)$

Winning probability: ...

Deterministic non-signaling strategies

Let's play SUDOKU!

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	$P(0,0,0,1)$	$P(0,0,1,0)$	$P(0,0,1,1)$
$(0,1)$	$P(0,1,0,0)$	$P(0,1,0,1)$	$P(0,1,1,0)$	$P(0,1,1,1)$
$(1,0)$	$P(1,0,0,0)$	$P(1,0,0,1)$	$P(1,0,1,0)$	$P(1,0,1,1)$
$(1,1)$	$P(1,1,0,0)$	$P(1,1,0,1)$	$P(1,1,1,0)$	$P(1,1,1,1)$

Winning probability: ...

Deterministic non-signaling strategies

Let's play SUDOKU!

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	$P(0,0,0,1)$	$P(0,0,1,0)$	$P(0,0,1,1)$
$(0,1)$	0	$P(0,1,0,1)$	$P(0,1,1,0)$	$P(0,1,1,1)$
$(1,0)$	0	$P(1,0,0,1)$	$P(1,0,1,0)$	$P(1,0,1,1)$
$(1,1)$	0	$P(1,1,0,1)$	$P(1,1,1,0)$	$P(1,1,1,1)$

Winning probability: ...

Deterministic non-signaling strategies

Let's play SUDOKU!

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	$P(0,0,0,1)$	$P(0,0,1,0)$	$P(0,0,1,1)$
$(0,1)$	0	$P(0,1,0,1)$	0	$P(0,1,1,1)$
$(1,0)$	0	0	$P(1,0,1,0)$	$P(1,0,1,1)$
$(1,1)$	0	0	0	$P(1,1,1,1)$

Winning probability: ...

Deterministic non-signaling strategies: Version 1

Let's play SUDOKU!

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	1	$P(0,0,1,0)$	$P(0,0,1,1)$
$(0,1)$	0	0	0	$P(0,1,1,1)$
$(1,0)$	0	0	$P(1,0,1,0)$	$P(1,0,1,1)$
$(1,1)$	0	0	0	$P(1,1,1,1)$

Winning probability: ...

Deterministic non-signaling strategies: Version 1

Let's play SUDOKU!

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	1	$P(0,0,1,0)$	$P(0,0,1,1)$
$(0,1)$	0	0	0	0
$(1,0)$	0	0	$P(1,0,1,0)$	$P(1,0,1,1)$
$(1,1)$	0	0	0	0

Winning probability: ...

Deterministic non-signaling strategies: Version 1.1

Let's play SUDOKU!

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	1	1	$P(0,0,1,1)$
$(0,1)$	0	0	0	0
$(1,0)$	0	0	0	$P(1,0,1,1)$
$(1,1)$	0	0	0	0

Winning probability: ...

Deterministic non-signaling strategies: Version 1.1

Let's play SUDOKU!

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	1	1	1
$(0,1)$	0	0	0	0
$(1,0)$	0	0	0	0
$(1,1)$	0	0	0	0

Winning probability: $3/4$

Deterministic non-signaling strategies: Version 1.2

Let's play SUDOKU!

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	1	0	$P(0,0,1,1)$
$(0,1)$	0	0	0	0
$(1,0)$	0	0	1	$P(1,0,1,1)$
$(1,1)$	0	0	0	0

Winning probability: ...

Deterministic non-signaling strategies: Version 1.2

Let's play SUDOKU!

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	1	0	0
$(0,1)$	0	0	0	0
$(1,0)$	0	0	1	1
$(1,1)$	0	0	0	0

Winning probability: $3/4$

Deterministic non-signaling strategies: Version 2.1

Let's play SUDOKU!

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	0	1	0
$(0,1)$	0	1	0	1
$(1,0)$	0	0	0	0
$(1,1)$	0	0	0	0

Winning probability: $3/4$

Deterministic non-signaling strategies: Version 2.2

Let's play SUDOKU!

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	1	0	0	0
$(0,1)$	0	1	0	0
$(1,0)$	0	0	1	0
$(1,1)$	0	0	0	1

Winning probability: $1/4$

Deterministic strategies

- There are 4x4=16 deterministic non-signaling strategies out of $4^4 = 256$

Deterministic strategies

- There are 4x4=16 deterministic non-signaling strategies out of $4^4 = 256$
- Alice/Bob have 4 local deterministic strategies: Constant 0, constant 1, identity, flip

Deterministic strategies

- There are 4x4=16 deterministic non-signaling strategies out of $4^4 = 256$
- Alice/Bob have 4 local deterministic strategies: Constant 0, constant 1, identity, flip
- Deterministic non-signaling strategies= Deterministic local strategies

Deterministic strategies

- There are 4x4=16 deterministic non-signaling strategies out of $4^4 = 256$
- Alice/Bob have 4 local deterministic strategies: Constant 0, constant 1, identity, flip
- Deterministic non-signaling strategies= Deterministic local strategies
- So non-signaling just means local? (NO!)

Local (random) strategies

- Convex combination of local deterministic strategies
  (Constant 0, constant 1, identity, flip)

Local (random) strategies

- Convex combination of local deterministic strategies
  (Constant 0, constant 1, identity, flip)
- Both Alice and Bob choses one local (random) strategy.

Local (random) strategies

- Convex combination of local deterministic strategies (Constant 0, constant 1, identity, flip)
- Both Alice and Bob choses one local (random) strategy.
- Mathematically: $P(a, b|x, y) = P(a|x)P(b|y)$

Local (random) strategies

- Convex combination of local deterministic strategies (Constant 0, constant 1, identity, flip)
- Both Alice and Bob choses one local (random) strategy.
- Mathematically: $P(a, b|x, y) = P(a|x)P(b|y)$
- This is not convex!

Strategies with shared randomness (hidden variable)

- Take the convex hull of the local random strategies!

Strategies with shared randomness (hidden variable)

- Take the convex hull of the local random strategies!
- Interpretation: Alex and Bob may share a random variable.

Strategies with shared randomness (hidden variable)

- Take the convex hull of the local random strategies!
- Interpretation: Alex and Bob may share a random variable.
- Again a polytope: convex hull of local deterministic strategies $\rightarrow$ (Spoiler!) Classical strategies

Strategies with shared randomness (hidden variable)

- Take the convex hull of the local random strategies!
- Interpretation: Alex and Bob may share a random variable.
- Again a polytope: convex hull of local deterministic strategies $\rightarrow$ (Spoiler!) Classical strategies
- Non-signaling $=$ classical? (NO!)

Strategies with shared randomness (hidden variable)

- Take the convex hull of the local random strategies!
- Interpretation: Alex and Bob may share a random variable.
- Again a polytope: convex hull of local deterministic strategies $\rightarrow$ (Spoiler!) Classical strategies
- Non-signaling = classical? (NO!)
- We have seen that (classical) random strategies won't help winning the game,
  deterministic strategies are the best...

Strategies with shared randomness (hidden variable)

- Take the convex hull of the local random strategies!
- Interpretation: Alex and Bob may share a random variable.
- Again a polytope: convex hull of local deterministic strategies $\rightarrow$ (Spoiler!) Classical strategies
- Non-signaling = classical? (NO!)
- We have seen that (classical) random strategies won't help winning the game,
  deterministic strategies are the best...
- ... so far

Non-signaling winning example: PR box

- Popescu-Rohrlich box:

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$1/2$	$1/2$	$1/2$	0
$(0,1)$	0	0	0	$1/2$
$(1,0)$	0	0	0	$1/2$
$(1,1)$	$1/2$	$1/2$	$1/2$	0

Non-signaling winning example: PR box

- Popescu-Rohrlich box:

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$1/2$	$1/2$	$1/2$	0
$(0,1)$	0	0	0	$1/2$
$(1,0)$	0	0	0	$1/2$
$(1,1)$	$1/2$	$1/2$	$1/2$	0

- Winning probability: $1 > 3/4 \Rightarrow$ this is not classical!

Non-signaling winning example: PR box

- Popescu-Rohrlich box:

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$1/2$	$1/2$	$1/2$	0
$(0,1)$	0	0	0	$1/2$
$(1,0)$	0	0	0	$1/2$
$(1,1)$	$1/2$	$1/2$	$1/2$	0

- Winning probability: $1 > 3/4 \Rightarrow$ this is not classical!
- So Non-signaling non-classical strategies exist, and they can improve the winning probability...

Non-signaling winning example: PR box

- Popescu-Rohrlich box:

$(a,b) \setminus (x,y)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$1/2$	$1/2$	$1/2$	0
$(0,1)$	0	0	0	$1/2$
$(1,0)$	0	0	0	$1/2$
$(1,1)$	$1/2$	$1/2$	$1/2$	0

- Winning probability: $1 > 3/4 \Rightarrow$ this is not classical!
- So Non-signaling non-classical strategies exist, and they can improve the winning probability...
- ... but what should Alice and Bob do?

Quantum strategies

- Alice and Bob share a (pure) state ψ.

Quantum strategies

- Alice and Bob share a (pure) state ψ.
- Upon receiving their inputs x and y they perform local measurements: M_x, N_y

Quantum strategies

- Alice and Bob share a (pure) state ψ.
- Upon receiving their inputs x and y they perform local measurements: M_x, N_y
- Then they send the measurement outcomes a and b.

Quantum strategies

- Alice and Bob share a (pure) state ψ.
- Upon receiving their inputs x and y they perform local measurements: M_x, N_y
- Then they send the measurement outcomes a and b.
- By the Born Rule : $P(a,b|x,y) = \langle\psi| \left(M_{x,a} \otimes N_{y,b}\right) |\psi\rangle$.

Quantum strategies

- Alice and Bob share a (pure) state ψ.
- Upon receiving their inputs x and y they perform local measurements: M_x, N_y
- Then they send the measurement outcomes a and b.
- By the Born Rule : $P(a,b|x,y) = \langle\psi| \left(M_{x,a} \otimes N_{y,b}\right) |\psi\rangle$.
- We also consider the convex combinations of these strategies.

Quantum strategies

- Quantum strategies are non-signaling, but larger than classical!

Quantum strategies

- Quantum strategies are non-signaling, but larger than classical!
- Quantum strategies are not a polytope!

Quantum strategies

- Quantum strategies are non-signaling, but larger than classical!
- Quantum strategies are not a polytope!
- Alice and Bob need an entangled state to improve on classical strategies!

Quantum strategies

- Quantum strategies are non-signaling, but larger than classical!
- Quantum strategies are not a polytope!
- Alice and Bob need an entangled state to improve on classical strategies!
- The best achievable winning rate of CHSH by quantum strategy is $\frac{2+\sqrt{2}}{4} \approx 0.854$ (PR is not quantum) !

Thanks!