

$$\sin^2 x + \cos^2 x = 1$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x \sin y = -\frac{1}{2} [\cos(x+y) - \cos(x-y)]$$

$$\sin x \cos y = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x+y) + \cos(x-y)]$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\text{arsinh } x = \ln \left( x + \sqrt{x^2 + 1} \right)$$

$$\text{arcosh } x = \ln \left( x + \sqrt{x^2 - 1} \right)$$

$$\text{artanh } x = \frac{1}{2} \ln \frac{1+x}{1-x}$$

$$\text{arcoth } x = \frac{1}{2} \ln \frac{x+1}{x-1}$$

$$(cf)' = cf'$$

$$(f \pm g)' = f' \pm g'$$

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$(f \circ g)' = (f' \circ g)g'$$

$$(x^\alpha)' = \alpha x^{\alpha-1}$$

$$(a^x)' = (\ln a)a^x$$

$$(\log_a x)' = \frac{1}{\ln a} \frac{1}{x}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

$$(\cot x)' = -\frac{1}{\sin^2 x} = -1 - \cot^2 x$$

$$(\arcsin x)' = -(\arccos x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = -(\operatorname{arccot} x)' = \frac{1}{1+x^2}$$

$$(\sinh x)' = \cosh x$$

$$(\cosh x)' = \sinh x$$

$$(\tanh x)' = \frac{1}{\cosh^2 x} = 1 - \tanh^2 x$$

$$(\coth x)' = -\frac{1}{\sinh^2 x} = 1 - \coth^2 x$$

$$(\text{arsinh } x)' = \frac{1}{\sqrt{x^2 + 1}}$$

$$(\text{arcosh } x)' = \frac{1}{\sqrt{x^2 - 1}}$$

$$(\text{artanh } x)' = \frac{1}{1-x^2}$$

$$(\text{arcoth } x)' = \frac{1}{1-x^2}$$

$$\int c f(x) dx = c \int f(x) dx$$

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

$$\int f'(ax+b) dx = \frac{1}{a} f(ax+b) + C$$

$$\int f^\alpha(x) f'(x) dx = \begin{cases} \frac{f^{\alpha+1}}{\alpha+1} + C & (\alpha \neq -1) \\ \ln |f(x)| + C & (\alpha = -1) \end{cases}$$

$$\int f(x) g'(x) dx = f(x) g(x) - \int f'(x) g(x) dx$$

$$t = \tan \frac{x}{2}$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\frac{dx}{dt} = \frac{2}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$u = \tanh \frac{x}{2}$$

$$\sinh x = \frac{2u}{1-u^2}$$

$$\frac{dx}{du} = \frac{2}{1-u^2}$$

$$\cosh x = \frac{1+u^2}{1-u^2}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} + \cdots$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots + \frac{x^{2n-1}}{(2n-1)!} + \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \cdots + \binom{\alpha}{n} x^n + \cdots$$

$$(1+x)^n \geq 1+nx$$

$$\det A = \sum_{i=1}^n (-1)^{i+j} a_{ij} \det A_{ij}$$

$$\binom{\alpha}{n} = \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}$$

$$\frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}} \leq \sqrt[n]{a_1 a_2 \cdots a_n} \leq \frac{a_1 + a_2 + \cdots + a_n}{n} \leq \sqrt{\frac{a_1^2 + a_2^2 + \cdots + a_n^2}{n}}$$

$$\begin{aligned} a_0 &+ \sum_{n=1}^{\infty} \left( a_n \cos \frac{2\pi n x}{T} + b_n \sin \frac{2\pi n x}{T} \right) \\ a_0 &= \frac{1}{T} \int_0^T f(x) dx \\ a_n &= \frac{2}{T} \int_0^T f(x) \cos \frac{2\pi n x}{T} dx \\ b_n &= \frac{2}{T} \int_0^T f(x) \sin \frac{2\pi n x}{T} dx \end{aligned}$$

$$\begin{aligned} (\mathcal{L}f)(z) &= \int_0^\infty f(x) e^{-zx} dx \\ (f * g)(x) &= \int_0^x f(s) g(x-s) ds \end{aligned}$$

$$h(x) \quad (\mathcal{L}h)(z)$$

$$1 \quad \frac{1}{z}$$

$$x^n \quad \frac{n!}{z^{n+1}}$$

$$e^{\alpha x} \quad \frac{1}{z-\alpha}$$

$$\cos bx \quad \frac{z}{z^2+b^2}$$

$$\sin bx \quad \frac{b}{z^2+b^2}$$

$$\cosh bx \quad \frac{z}{z^2-b^2}$$

$$\sinh bx \quad \frac{b}{z^2-b^2}$$

$$\begin{aligned} \text{grad } f &= \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k} \\ \text{div } \mathbf{u} &= \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \\ \text{rot } \mathbf{u} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u_x & u_y & u_z \end{vmatrix} = \left( \frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z} \right) \mathbf{i} + \left( \frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} \right) \mathbf{j} + \left( \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \right) \mathbf{k} \end{aligned}$$

$$D\mathbf{u} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

$$\begin{aligned} \text{rot grad } f &= 0 \\ \text{div rot } \mathbf{u} &= 0 \\ \text{grad } cf &= c \text{grad } f \\ \text{div } c\mathbf{u} &= c \text{div } \mathbf{u} \\ \text{rot } c\mathbf{u} &= c \text{rot } \mathbf{u} \\ \text{grad}(f \pm g) &= \text{grad } f \pm \text{grad } g \\ \text{div}(\mathbf{u} \pm \mathbf{v}) &= \text{div } \mathbf{u} \pm \text{div } \mathbf{v} \\ \text{rot}(\mathbf{u} \pm \mathbf{v}) &= \text{rot } \mathbf{u} \pm \text{rot } \mathbf{v} \\ \text{grad}(fg) &= (\text{grad } f)g + f \text{grad } g \\ \text{div}(f\mathbf{u}) &= (\text{grad } f) \cdot \mathbf{u} + f \text{div } \mathbf{u} \\ \text{rot}(f\mathbf{u}) &= (\text{grad } f) \times \mathbf{u} + f \text{rot } \mathbf{u} \\ \text{grad}(\mathbf{u} \cdot \mathbf{v}) &= D\mathbf{u}(\mathbf{v}) + D\mathbf{v}(\mathbf{u}) + \mathbf{u} \times \text{rot } \mathbf{v} + \mathbf{v} \times \text{rot } \mathbf{u} \\ \text{rot}(\mathbf{u} \times \mathbf{v}) &= D\mathbf{u}(\mathbf{v}) - D\mathbf{v}(\mathbf{u}) + (\text{div } \mathbf{v}) \cdot \mathbf{u} - (\text{div } \mathbf{u}) \cdot \mathbf{v} \\ \text{div}(\mathbf{u} \times \mathbf{v}) &= (\text{rot } \mathbf{u}) \cdot \mathbf{v} - \mathbf{u} \cdot (\text{rot } \mathbf{v}) \end{aligned}$$

$$\begin{aligned} \int_C f ds &= \int_a^b f(\mathbf{r}(t)) |\dot{\mathbf{r}}(t)| dt \\ \int_C \mathbf{u} \cdot d\mathbf{r} &= \int_a^b \mathbf{u}(\mathbf{r}(t)) \cdot \dot{\mathbf{r}}(t) dt \\ \iint_S f dA &= \iint_D f(\mathbf{r}(u, v)) \left| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right| du dv \\ \iint_S \mathbf{u} \cdot d\mathbf{A} &= \iint_D \mathbf{u}(\mathbf{r}(u, v)) \cdot \left( \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right) du dv \\ \iiint_V f dV &= \iiint_D f(\mathbf{r}(u, v, w)) \left| \frac{\partial \mathbf{r}}{\partial u} \frac{\partial \mathbf{r}}{\partial v} \frac{\partial \mathbf{r}}{\partial w} \right| du dv dw \\ \int_C \text{grad } f \cdot d\mathbf{r} &= f(\mathbf{r}(b)) - f(\mathbf{r}(a)) \\ \iint_S \text{rot } \mathbf{u} \cdot d\mathbf{A} &= \int_{\partial S} \mathbf{u} \cdot d\mathbf{r} \\ \iiint_V \text{div } \mathbf{u} dV &= \iint_{\partial V} \mathbf{u} \cdot d\mathbf{A} \\ \iint_{\partial V} f \text{grad } g \cdot d\mathbf{A} &= \iiint_V (f \Delta g + \text{grad } f \cdot \text{grad } g) dV \\ \iiint_V (f \Delta g - g \Delta f) dV &= \iint_{\partial V} (f \text{grad } g - g \text{grad } f) \cdot d\mathbf{A} \end{aligned}$$

$$h(x) \quad (\mathcal{L}h)(z)$$

$$x^n f(x) \quad (-1)^n (\mathcal{L}f)^{(n)}(z)$$

$$e^{\alpha x} f(x) \quad (\mathcal{L}f)(z-\alpha)$$

$$f^{(n)}(x) \quad z^n (\mathcal{L}f)(z) - z^{n-1} f(0) - z^{n-2} f'(0) - \cdots - f^{(n-1)}(0)$$

$$(f * g)(x) \quad (\mathcal{L}f)(z) (\mathcal{L}g)(z)$$

$$P(x,y) + Q(x,y)y' = 0$$

$$\ln |M(x)| = \int \frac{\frac{\partial P(x,y)}{\partial y} - \frac{\partial Q(x,y)}{\partial x}}{Q(x,y)} dy$$

$$\ln |M(y)| = \int \frac{\frac{\partial Q(x,y)}{\partial x} - \frac{\partial P(x,y)}{\partial y}}{P(x,y)} dy$$