Combinatorial and discrete geometry – exam topics

2025/26 autumn

1 Convex polytopes and polyhedral sets.

conical hull, H- and V-, polyhedral sets and polytopes, face, dimension, algebra of compact convex sets, valuation, Euler characteristic, Euler's formula (proof), face lattice, properties of the face lattice, combinatorially equivalent and dual polytopes

2 Upper bound theorem

f-vector, simple and simplicial polytopes, properties of simple polytopes (proof), h-vector with respect to a linear functional, relation between h-vector and f-vector (proof), Dehn–Sommerville equations (proof), relation between h-vector of a simple polytope and that of its facets, upper bound on h-vector of a simple polytope (proof), upper bound theorem (proof), cyclic polytopes and their faces (proof)

3 Graphs of polytopes and convex hull

vertex connectivity, graph of a polytope, Balinski's theorem (proof), SP-reduction, ΔY operations, geometric realizability of simple ΔY reduction (proof), graph minor, Steinitz's theorem (main ideas of proof), convex hull computation, Andrew's algorithm (proof)

4 Incidence problems

point–line incidences, lower bound (proof), graph drawing, crossing number, crossing number theorem (proof), Szemerédi–Trotter theorem (proof), unit distances, upper bound, representing positive integers as sums of two squares, prime counting in residue classes, lower bound on unit distances (proof), upper bound on the number of unit-area triangles (proof)

5 Arrangements

arrangements of hyperplanes, number of faces (proof), level, Clarkson's theorem (proof), arrangements of segments, Davenport–Schinzel sequence, $O(n \log n)$ upper bound (proof), $\omega(n)$ lower bound (proof), Ackermann function and the growth of the maximum complexity

6 Ramsey-type results

points in convex position, Klein's problem, small cases and finiteness of $M(k)$ (proof), Erdős–Szekeres theorem (proof), $M(k)$ for $k \leq 5$ (proof), holes, upper bound on $M'(5)$, 7-holes and Horton sets, Horton's theorem (proof), convex sets in convex position, tournaments, Pach–Tóth theorem (proof)

7 Antipodal sets

antipodal set, sets without obtuse triangles, central symmetrization, width, properties of central symmetrization (proof), overlapping and touching convex bodies, Minkowski lemma (proof), Danzer–Grünbaum theorem (proof), Erdős–Füredi theorem (proof), homothetic images, Bezdek–Connelly theorem (proof), Naszódi theorem (proof)

8 Hadwiger number

Hadwiger number, upper bound for convex bodies in $\mathbb{R}^d$ (proof), Grünbaum theorem, topological disks and star-convex sets, kernel, Bezdek–Kuperberg–Kuperberg conjecture, Cheong–Lee theorem (construction, proof), norms and centralsymmetric convex bodies, distance, distance estimates for disjoint and overlapping translates, perimeter, theorems of Gołąb and Fáry–Makai, upper bound on the Hadwiger number of centralsymmetric star-convex disks (proof)

9 Borsuk's partition problem

Borsuk number, upper bound method, theorems of Lenz and Knast (proof), superlinear lower bound (construction, proof), inclusion in regular hexagon (proof), sets of constant width, properties of constant-width convex bodies (proof), r-ball bodies, properties of generated r-ball body (proof), inclusion of compact sets in sets of constant width (proof), Boltyanski's theorem

10 Combinatorial topology

geometric and abstract simplicial complexes, geometric realization, canonical realization, uniqueness (proof), face poset, order complex, barycentric subdivision, Sperner lemma (proof), Brower fixed point theorem (proof)