# Convex geometry
# Assignment

**Problem 1.** *Prove that if $K \subseteq \mathbb{R}^n$ is closed, convex and not bounded, then for every $p \in K$, $K$ contains a half line starting at $p$.*

**Problem 2.** *Let $K \subseteq \mathbb{R}^n$ be a closed, convex set. Prove that if $K$ contains a half line $L$ starting at $p$, then for any $q \in K$, $K$ contains the half line starting at $q$ and having the same direction as $L$.*

**Problem 3.** *Let $K \subseteq \mathbb{R}^n$ be the convex hull of the points whose every coordinate is $1$ or $-1$, i.e. $K$ is an origin-symmetric cube of edge length $2$. Define $K$ as the intersection of finitely many closed half spaces.*

**Problem 4.** *Let $e_1, \ldots, e_n$ be the standard basis vectors of $\mathbb{R}^n$. Let $K = \operatorname{conv}\{\pm e_1, \pm e_2, \ldots, \pm e_n\}$. Define $K$ as the intersection of finitely many closed half spaces.*

**Problem 5.** *Let $X \subset \mathbb{R}^n$ be a compact set. Let $H$ be an open half space containing $X$. Prove that then there is a closed half space $H_0 \subset H$ such that $X \subset H_0$. Is this statement true if $X$ is not necessarily closed?*

**Problem 6.** *A compact, convex set $K \subset \mathbb{R}^n$ is called a set of* constant width *$d$, if for any unit vector $u$, the distance of the pair of supporting hyperplanes perpendicular to $u$ is $d$. Prove that a compact, convex set $K$ is of constant width $d$ if and only if $h_K(u) + h_K(-u) = d$ for every unit vector $u$.*

**Problem 7.** *Let $T$ be a regular triangle of edge length $a > 0$. The intersection of the three closed disks of radius $a$ centered at the vertices of $T$ is called a* Reuleaux triangle. *Prove that this set is a set of constant width $a$.*

**Problem 8.** *Let $R$ be the Reuleaux triangle defined in the previous problem. Prove or disprove that the set $R + \lambda B^2$ is a set of constant width for every $\lambda \in \mathbb{R}$. ($B^2$ is the closed unit disk centered at the origin.)*

**Problem 9.** *Let $T$ be a regular tetrahedron in $\mathbb{R}^3$ of edge length $a$. Let $R$ denote the intersection of the four closed unit balls centered at the vertices of $T$. Prove that $R$ is not a set of constant width $a$.*