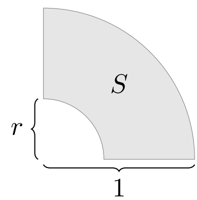
1. An $m \times n$ matrix $A = (a_{ij})_{\substack{i=1,\dots,m\\ j=1,\dots,n}}$ is called column stochastic if it has nonnegative entries and every column sum is 1, i.e., $\sum_{i=1}^{m} a_{ij} = 1$ for all $j = 1, \dots, n$. Prove that the set of $m \times n$ column stochastic matrices is compact and convex. (5 points)

2. Recall that the kernel of a set $S \subseteq \mathbb{R}^n$ is the set of points x with the property that $[x, y] \subseteq S$ holds for any $y \in S$. Find the kernel of the closed set S below, bounded by two perpendicular straight line segments and two circular arcs, where $r \in (0, 1)$. (5 points)



3. Prove that the Minkowski sum of an open set and an arbitrary set is open. (5 points)

4. Let $K \subseteq \mathbb{R}^n$ be compact and convex, and $A : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. Prove that $h_{A(K)}(x) = h_K(A^T(x))$ for all $x \in \mathbb{R}^n$. Use this to find the support function of an ellipse in the plane, centered at the origin, with half axes of length 2 and 1 parallel to the x and y axes, respectively. (Recall that the linear transformation A^T is characterised by the equality $\langle x, Ay\rangle = \langle A^T x, y\rangle$ for all $x, y \in \mathbb{R}^n$.) (5 points)