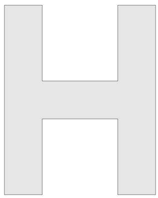
May 10, 2024 Convex geometry **Midterm 2**

1. Let K be the convex hull of $\{(0,0),(1,0),(1,1),(0,1)\}$ and let L the convex hull of $\{(0,0),(1,0),(0,1)\}$. For $\alpha, \beta \geq 0$ find the area of $\alpha K + \beta L$. (5 points)

2. Let $K \subseteq \mathbb{R}^n$ be a nonempty convex set that can be written as the intersection of finitely many closed balls. Prove that every boundary point of K is an exposed point. (5 points)

3. Express the indicator function of the (closed) shape below as a linear combination of indicator functions of compact convex sets. Use this to find its Euler characteristic. (5 points)



4. Suppose that the vectors $x_1, \ldots, x_k \in \mathbb{R}^n$ satisfy $\|x_i\| = \|x_j\|$ for all $i, j \in \{1, \ldots, k\}$. Show that $M = \{x_1, x_2, \ldots, x_k\}$ is the minimal representation of $\operatorname{conv}\{x_1, x_2, \ldots, x_k\}$. (5 points)