

Differenciálformák

$$\begin{aligned}\alpha \wedge (\beta \wedge \gamma) &= (\alpha \wedge \beta) \wedge \gamma \\ (\alpha_1 + \alpha_2) \wedge \beta &= \alpha_1 \wedge \beta + \alpha_2 \wedge \beta \\ \alpha \wedge \beta &= (-1)^{kl} \beta \wedge \alpha \\ \varphi^*(\alpha \wedge \beta) &= \varphi^*\alpha \wedge \varphi^*\beta \\ (\varphi \circ \psi)^*\alpha &= \psi^*\varphi^*\alpha\end{aligned}$$

Külső derivált

$$\begin{aligned}\mathbf{i}_X df &= X(f) \\ d d\alpha &= 0 \\ d(\alpha \wedge \beta) &= d\alpha \wedge \beta + (-1)^k \alpha \wedge d\beta \\ \varphi^* d\alpha &= d\varphi^*\alpha\end{aligned}$$

Belső szorzás (behelyettesítés)

$$\begin{aligned}\mathbf{i}_{fX}\alpha &= f\mathbf{i}_X\alpha = \mathbf{i}_X(f\alpha) \\ \mathbf{i}_X\mathbf{i}_X\alpha &= 0 \\ \mathbf{i}_X(\alpha \wedge \beta) &= \mathbf{i}_X\alpha \wedge \beta + (-1)^k \alpha \wedge \mathbf{i}_X\beta \\ \varphi^*\mathbf{i}_X\alpha &= \mathbf{i}_{\varphi^*X}\varphi^*\alpha\end{aligned}$$

Lie-derivált

$$\begin{aligned}\mathcal{L}_X f &= (df)(X) \\ \mathcal{L}_{\varphi_*X}\varphi^*f &= \varphi_*\mathcal{L}_X f \\ \mathcal{L}_X(fg) &= (\mathcal{L}_X f)g + f(\mathcal{L}_X g) \\ \mathcal{L}_{X_1+X_2}f &= \mathcal{L}_{X_1}f + \mathcal{L}_{X_2}f \\ \mathcal{L}_X(f+g) &= \mathcal{L}_X f + \mathcal{L}_X g \\ \mathcal{L}_{fX}(g) &= f\mathcal{L}_X g \\ \mathcal{L}_X\alpha &= \mathbf{d}\mathbf{i}_X\alpha + \mathbf{i}_X d\alpha \\ \mathcal{L}_X(\alpha \wedge \beta) &= \mathcal{L}_X\alpha \wedge \beta + \alpha \wedge \mathcal{L}_X\beta \\ \mathcal{L}_{[X,Y]} &= [\mathcal{L}_X, \mathcal{L}_Y] \\ [X, Y] &= \mathcal{L}_X Y - \mathcal{L}_Y X \\ 0 &= [X, [Y, Z]] + [Y, [Z, X]] + [Z, [X, Y]] \\ \mathcal{L}_{fX}\alpha &= f\mathcal{L}_X\alpha + df \wedge \mathbf{i}_X\alpha \\ \mathcal{L}_{[X,Y]}\alpha &= \mathcal{L}_X\mathcal{L}_Y\alpha - \mathcal{L}_Y\mathcal{L}_X\alpha \\ \mathbf{i}_{[X,Y]}\alpha &= \mathcal{L}_X\mathbf{i}_Y\alpha - \mathbf{i}_Y\mathcal{L}_X\alpha \\ \mathcal{L}_X d\alpha &= d\mathcal{L}_X\alpha \\ \mathcal{L}_X\mathbf{i}_X\alpha &= \mathbf{i}_X\mathcal{L}_X\alpha\end{aligned}$$

f, g sima függvény, X, Y, Z vektormező, $\alpha, \alpha_1, \alpha_2$ k -formák, β l -forma, γ differenciálforma, φ, ψ sima leképezések (ha vektormező is szerepel, akkor diffeomorfizmusok).